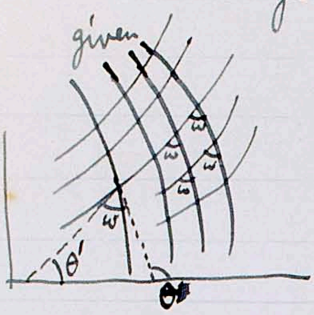
Chapter XX

Some applications

90. Isogonal trajectories 等角交截線



$\operatorname{tg}\theta = \dfrac{dy}{dx}$ 与ヘラレタ曲線群.

$\theta' = \theta \pm \omega.$ $\qquad \operatorname{tg}\theta' = \dfrac{\operatorname{tg}\theta - \operatorname{tg}\omega}{1+\operatorname{tg}\omega\operatorname{tg}\theta} = \dfrac{\frac{dy}{dx} - \operatorname{tg}\omega}{1+\operatorname{tg}\omega\frac{dy}{dx}}.$

故ニ元ノ曲線群ノ微分方程式ガ

$$f\left(\frac{dy}{dx}, x, y\right) = 0$$

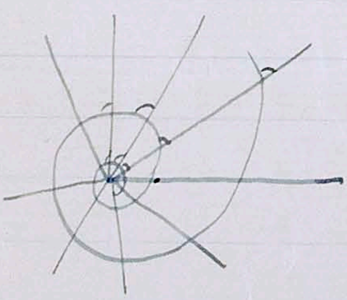
トスレバ, isogonal traj. ノ微分方程式ハ

$$f\left(\frac{\frac{dy}{dx} - \operatorname{tg}\omega}{1+\operatorname{tg}\omega\frac{dy}{dx}}, x, y\right) = 0.$$

特ニ $\omega = \frac{\pi}{2}$ トスレバ ~~$\operatorname{tg}\omega = \infty$~~ $\longrightarrow$ $\operatorname{tg}\theta' \cdot \operatorname{tg}\theta = -1.$ $\quad \operatorname{tg}\theta' = -\dfrac{1}{\frac{dy}{dx}},$

B. 1.

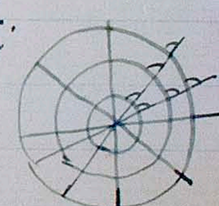
$$f\left(-\frac{1}{\frac{dy}{dx}}, x, y\right) = 0. \qquad \text{orthogonal traj.}$$



$y = cx,$ $\quad$ コノ dif. eq. ハ $\dfrac{dy}{dx} = c$ $\quad \dfrac{dy}{dx} = \dfrac{y}{x}.$

故ニ ~~isogo~~ orth. traj. ハ $\qquad -\dfrac{1}{\frac{dy}{dx}} = \dfrac{y}{x}.$

$x\,dx + y\,dy = 0.$ $\qquad x^2 + y^2 = \text{const}.$



一般ノ場合ニハ

$$\frac{\frac{dy}{dx} - \operatorname{tg}\omega}{1+\operatorname{tg}\omega\frac{dy}{dx}} = \frac{y}{x}. \qquad \left(-1 + \frac{y}{x}\operatorname{tg}\omega\right)\frac{dy}{dx} + \left(\frac{y}{x} + \operatorname{tg}\omega\right) = 0.$$

homog.

$y = vx$

$\dfrac{dy}{dx} = v + x\dfrac{dv}{dx}.$

$$\operatorname{tg}\omega \cdot \frac{dx}{x} = \left(\frac{1}{1+v^2} - \frac{v}{1+v^2}\operatorname{tg}\omega\right)dv.$$

$$\operatorname{arctg} v - \frac{1}{2}\log(1+v^2)\cdot\operatorname{tg}\omega = \operatorname{tg}\omega\cdot\log x + \log c.$$

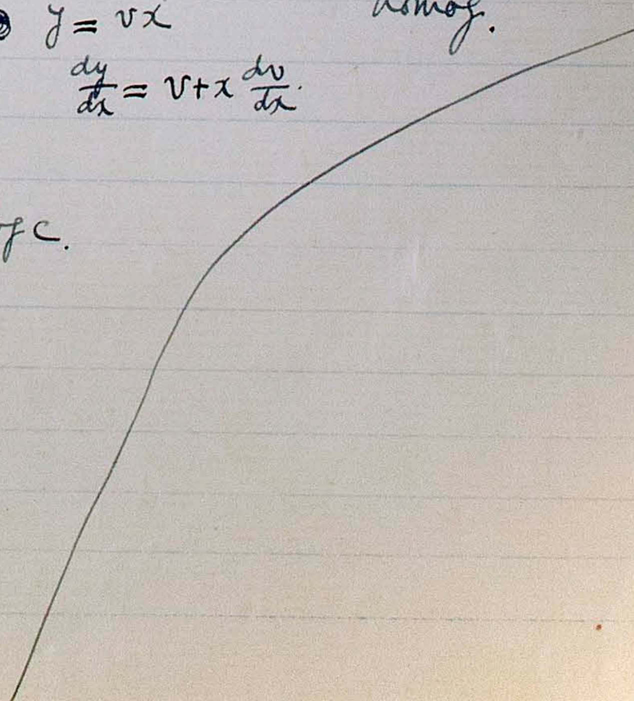
$$\operatorname{arctg} v = \log c\left(x\sqrt{1+v^2}\right)^{\operatorname{tg}\omega}$$

$$\operatorname{arctg}\frac{y}{x} = \log c\left(\sqrt{x^2+y^2}\right)^{\operatorname{tg}\omega}$$

$\theta = \log c\left(r^{\operatorname{tg}\omega}\right),$

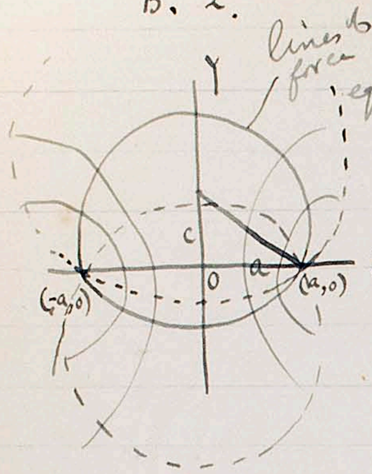
$c\,r^{\operatorname{tg}\omega} = e^{\theta}$ $\qquad r = \left(\dfrac{1}{c}e^{\theta}\right)^{\frac{1}{\operatorname{tg}\omega}}$

$r = A e^{\theta\cot\omega}$ $\qquad$ log spiral



電磁気 potentiel

B. 2.



$x^2+(y-c)^2=a^2+c^2.$

$x^2+y^2-2cy=a^2.$

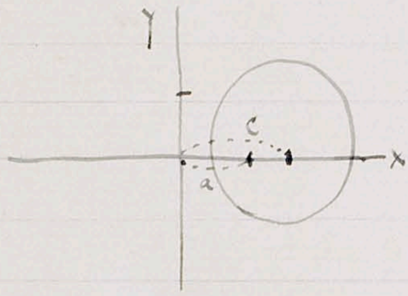
$\frac{x^2+y^2-a^2}{y}=2c.$

$y\left(2x+2y\frac{dy}{dx}\right)-\frac{dy}{dx}(x^2+y^2-a^2)=0.$

$\cancel{2xy+y^2\frac{dy}{dx}-x^2}$ $\quad 2xy-\frac{dy}{dx}(x^2-y^2-a^2)=0.$

orthog. traj. 〃 $\quad 2xy+\frac{1}{\frac{dy}{dx}}(x^2-y^2-a^2)=0.$

$2yx-\frac{dx}{dy}(y^2-x^2+a^2)=0.$



x	y	a
y	x	$\sqrt{-a}$

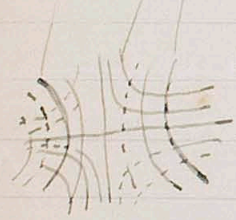
$y^2+(x-c)^2=c^2-a^2.$

$\cancel{y^2+(x-c)^2=}$ $\quad y^2+x^2-2cx=-a^2.$

流体力学.

等速線.

流線

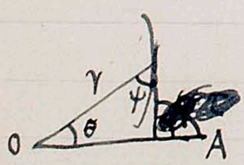


Aufgaben.

1. $x^2+y^2=2cx$, orthog. traj.
2. $y=e^x+c$, 3. $y=ax^n$, 4. $\frac{x^2}{a^2}+\frac{y^2}{a^2-c^2}=1$, orthog.
5. $xy=c$ orthogonal.

General exercises.

1.

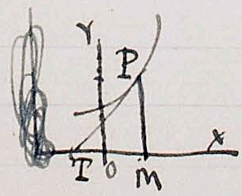


$\psi=n\theta$

(n, const)

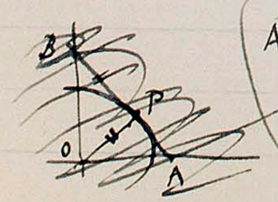
$\boxed{\frac{r}{\frac{dr}{d\theta}}=\tan n\theta}$ $\quad r^n=a^n \sin n\theta$

3.



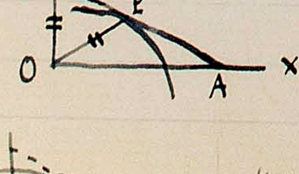
$\overline{TM}=\cancel{x+y}$ $OM+PM$ $\quad y=Ce^{\frac{x}{y}}$

2.



AB $\quad Y-y=\frac{dy}{dx}(X-x).$

$X=0, \quad Y=y-x\frac{dy}{dx}.$



$OB=OP$

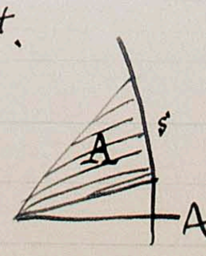
+ ~ 曲線.

$y-x\frac{dy}{dx}=\sqrt{x^2+y^2}.$

$y+\sqrt{x^2+y^2}=C.$

$x^2=-2Cy+C^2$

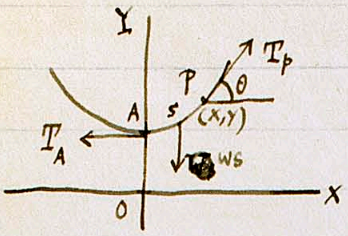
4.



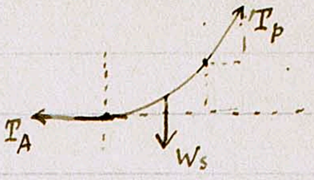
$A=ks.$

91. Catenary.

等質ナル糸ノ両端ヲ固定ス.



$\widehat{AP}=s$, 糸ノ単位ノ長サノ重サヲ w トスレバ, $\widehat{AP}$ ノ重サハ ws.



コノ三ツノ力ガ平衡スルタメノ條件

$T_p\cos\theta - T_A = 0,\quad T_p\sin\theta - ws = 0.$

T_A ハ const. ナレバ $\dfrac{T_A}{w} = a$ トオケバ:

$(T_p\cos\theta - aw = 0.$

$$\therefore\quad a\,tg\,\theta = s.$$

$$a\frac{dy}{dx} = \int_0^x \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx.$$

$\dfrac{dy}{dx} = p$ トオケバ:

$$ap = \int_0^x \sqrt{1+p^2}\,dx \qquad a\frac{dp}{dx} = \sqrt{1+p^2} \qquad \frac{dp}{\sqrt{1+p^2}} = \frac{dx}{a}$$

$$\log\left(p+\sqrt{1+p^2}\right) = \frac{x}{a} + c.$$

$x=0$ ノトキ $p=0$ ナルナレバ $c=0$.

$$p+\sqrt{1+p^2} = e^{\frac{x}{a}},\qquad p-\sqrt{1+p^2} = \frac{p-(1+p^2)}{p+\sqrt{1+p^2}} = -\frac{1}{p+\sqrt{1+p^2}} = -e^{-\frac{x}{a}}.$$

$$\therefore\quad p = \frac{1}{2}\left(e^{\frac{x}{a}} - e^{-\frac{x}{a}}\right) = \frac{dy}{dx}.$$

$$y = \frac{1}{2a}\left(e^{\frac{x}{a}} + e^{-\frac{x}{a}}\right) + C'$$