On the ~~theory~~ theory of time-correlation.

Poynting 1884
Hooker 1901
March 1905

W. M. Persons 1919, Review of economic statistics

季節的 / 変化
{ elimination of seasonal variation
〃 of secular trend

cyclic variation ニヲモ統スル

time lag ヲモ考ヘル

{ 結婚率 季節変化 不変
輸出入 〃 異〃 増加

nonsense non-sense

business forecasting || 財界予見
barometer

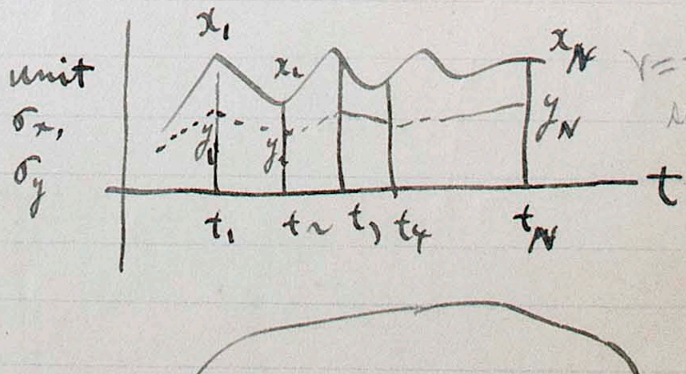
Davies — 蜷川
小倉
野村証券株式
会社

$x_1 \quad x_2 \quad \cdots \quad x_N \qquad M_x$

$y_1 \quad y_2 \quad \cdots \quad y_N \qquad M_y$

$$\sigma_x = \sqrt{\frac{1}{N}\sum_k (x_k - M_x)^2}$$

$$\sigma_y = \sqrt{\frac{1}{N}\sum_k (y_k - M_y)^2}$$



$r = +1$

$r = -1$ symmetric

$$r_{x,y} = \frac{\sum_k (x_k - M_x)(y_k - M_y)}{N \cdot \sigma_x \sigma_y}$$

$r = 0$ orthogonal

$\int x(t)\, y(t)\, dt = 0$

"Student", Elimination of spurious correlation, etc.
Biometrika, 10 (1914), p. 179.

I. $\begin{array}{c|ccccc} x & x_1 & x_2 & x_3 & \cdots & x_N \\ y & y_1 & y_2 & y_3 & \cdots & y_N \end{array}$ at random $\boxed{\text{mean} = 0}$

$$\frac{\sum}{N}(x_1-x_2)^2 = \frac{\sum}{N}(x_1^2 - 2x_1x_2 + x_2^2) = \frac{1}{N}(x_1^2+\cdots+x_{N-1}^2) + \frac{1}{N}(x_2^2+\cdots+x_N^2)$$
$$- \frac{2}{N}\sum x_1x_2$$

$$\sum x_1x_2 = 0 = x_1x_2 + x_2x_3 + \cdots + x_{N-1}x_N$$ mutually at random.

$$\frac{1}{N}(x_1^2+\cdots+x_{N-1}^2) = \frac{1}{N}(x_2^2+\cdots+x_N^2) = \frac{1}{N}(x_1^2+x_2^2+\cdots+x_N^2)$$

Nガ極メテ大ナレバ.

$$\therefore \quad \sigma^2_{\Delta x} = 2\sigma_x^2,$$

mean $= \frac{1}{N}(x_1+x_2+\cdots+x_N) = 0.$
$$\frac{1}{N}\left[(x_1-x_2)+(x_2-x_3)+\cdots+(x_{N-1}-x_N)\right] = \frac{x_1-x_N}{N} = 0.$$

同様ニ $$\sigma^2_{\Delta y} = 2\sigma_y^2,$$

$$(x_1-x_2)(y_1-y_2) = \overline{\overline{x_1y_1}} - x_2y_1 - x_1y_2 + \overline{\overline{x_2y_2}}$$

$$\frac{\sum}{N}(x_1-x_2)(y_1-y_2) = \frac{2}{N}\sum x_1y_1$$ $\overset{\shortparallel}{0}$ mutually at random.

$$r_{\Delta x, \Delta y} = \frac{\frac{\sum}{N}(x_1-x_2)(y_1-y_2)}{\sigma_{\Delta x}\sigma_{\Delta y}} = \frac{\sum x_1y_1}{N\sigma_x\sigma_y} = r_{x,y}.$$

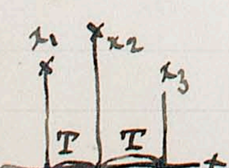
$r_{\Delta^2 x, \Delta^2 y} =$

$$\therefore \quad r_{\Delta_n x, \Delta_n y} = r_{x,y}$$

II. 之ニハ x, y ガ at random ニ[アラザル]場合. $\left\{ \begin{array}{l} x_1 = X_1 + \overset{n}{P}(t_1), \\ x_2 = X_2 + \overset{n}{P}(t_2), \end{array} \right.$
t ノ polynomial

$$x_1 = X_1 + bt_1 + ct_1^2 + \cdots t_1^n, \quad x_2 = X_2 + bt_2 + ct_2^2 + \cdots, t_1^n$$

$X_1, X_2, \cdots$ ハ time ニ independent.



$$y_1 = Y_1 + b't_1 + c't_1^2 + \cdots, \quad y_2 = Y_2 + b't_2 + c't_2^2 + \cdots,$$

$y_1 = Y_1 + \overset{n}{Q}(t_1)$
$y_2 = Y_2 + \overset{n}{Q}(t_2), \ldots$

$T = t_2 - t_1$

$$(x_1-x_2) = (X_1-X_2) - bT - cT(t_1+t_2) - \cancel{dT(t_1^2+t_1t_2+t_2^2)+\cdots}$$

$$\cancel{\underset{\Delta x}{(x_1-x_2)} = \underset{\Delta X}{(X_1-X_2)} - (bT + cT^2 + \cdots) - t_1(2cT + 3dT^2 + \cdots) - t_1^2(3dT + \cdots) + \cdots}$$

t ノ highest power ガ[一ツ減ル].

$$x_1 - x_2 = X_1 - X_2 + \overset{n-1}{P}(t_1, T).$$
$$\Delta x = \Delta X + \overset{n-1}{P}(t, T).$$

故ニ此 operat' ヲ幾回モ繰返セバ:

$$\Delta_n x = \Delta_n X + \text{const.}$$
$$\Delta_n y = \Delta_n Y + \text{const.}$$

random variable independent of t.

$$\therefore \quad r_{\Delta_n x, \Delta_n y} = r_{\Delta_n X, \Delta_n Y} = r_{X, Y}.$$
$$= r_{\Delta_{n+1} x, \Delta_{n+1} y}$$
$$= \cdots$$

O. Anderson, Biometrika, 10 (1914).

M. Cave and K. Pearson, Biometrika 10 (1914, ~~1915~~), p. 340

Numerical illustrations of the variate difference correlation method.

実際問題ニハ Nガ大ナラザル場合アリ. コレモ此場合ニモ此方法ハ有効カナリ.

$x_1, x_2, \cdots$ ハ平均カラ引イタモノ

1885–1912 (N=28) 伊太利ノ統計

	タバコ コーヒー 消費額	タバコ 消費額ト 貯蓄銀行ノ利子
0th difference (quantities)	0.955	0.984
1st diff.	0.653	0.766
2nd diff.	0.319	−0.044
3	0.349	−0.327
4	0.358	−0.380
5	0.344	−0.402
6	0.326	−0.431

W. M. Persons, Quarterly Publication of Amer. Statist. Association. 1917

(Rietz, Handbook of math. Statistics) 1924.

mutually random ノ仮定ハ non sense ナリ

此方法 (variate difference method) ハ simplest problem ニサヘモ

適用スベカラザル仮定~~ヲ…~~ノ上ニ立ツモノニシテ, time series ノ correlat ノ問題ニ對シテハ of little or no value デアル.

Bowley, two refined for ordinary observations. (1920).

G. U. Yule, On the time-correlation problem, Journal of the Royal Statistical Society, 84 (1921), p.497
30 ~~page~~ 頁. (外ニ 12頁ノ discussion アリ).

$$x = X + \phi(t)$$

〃random residual.

1. ヨシヤ random residual ガ isolate サレルモノトシテモ, ソレヲ比較シテ何ガ解ルノカ. 各々ノ目的ハ different durations ノ oscillations ヲ isolate スルニアル. time ニ関係ナキモノガ何ニナルノカ.

2. 果シテアノ方法デ random residual ヲワケ得ルカ. $\phi(t)$ ハ polynomial デナク, 寧ロ harmonic funct. ニアラズヤ. 若シ harmonic f. ナリトセバ, difference ハ無意味ノ場合アリ.

	Δ_1	Δ_2
0	+1	−2
+1	−1	0
0	−1	+2
−1	+1	0
0	+1	−2
+1	−1	
0		

Δ ハ ダンダンニ 大キクナル.

實際 Cave, Pearson ノ示セル伊太利ノ例ノ中ニハ Δ ガ ダンダンニ大キクナル場合モアル.

コレ polynomial ニアラザル証ナリ.

[伊太利ノ中 ナカバヨ ト 兒童貯蓄銀行ノ例ノ如キハ, 普通ノマヽニアラズシテ他ノ事情ニテ説明モ出来ルガ, 一ツハ 時ノ経過 他ノ理由モアル. ヨイ例ニアラズ]

Discussion

Yule ニ賛成 — variate difference method ニモ 価値ヲ認メタル

Edgeworth — Greenwood

Brownlee — Stamp

Trachtenberg

K. Pearson an E. M. Elderton, Biometrika 14 (1923).

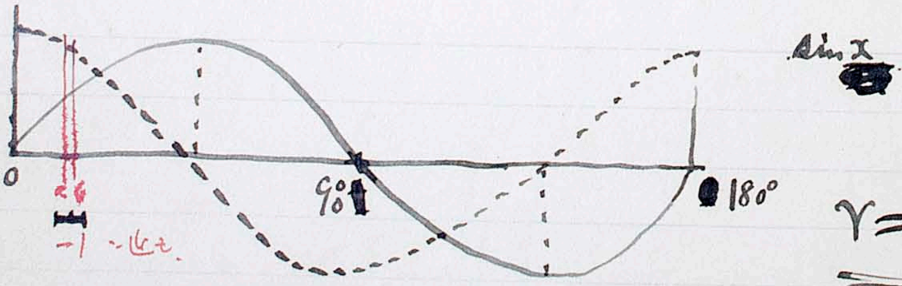
1. Persons ノ批判 mutually random $\Sigma x_1 x_2 = 0$ ノ不可能ハ正シ。コレヲヨリヌキノ仮定デ置換ヘテ、複雑ナ形ニ到達ス。

2. Yule ノ periodic terms ニ関スル批判ハ誤リ。根本ヨリ長イ計算デモ證明出来ル。

結論：
Variate difference method ハ元ノモノト大ニ改良サレシモノハ價値アル。

Mills, Statistical methods (1925).
Yule, Why do we sometimes get nonsense-correlations between time-series? J. of Royal Statistical Society, 1926, 1.

1866-1911 總テノ結婚者ニ對スル Church marriage ノ比。
1000人ニ對スル死亡者。
$r = +0.95$ $p.e = 0.147$



$r=0$ ノ場合!!