Biomachematics.

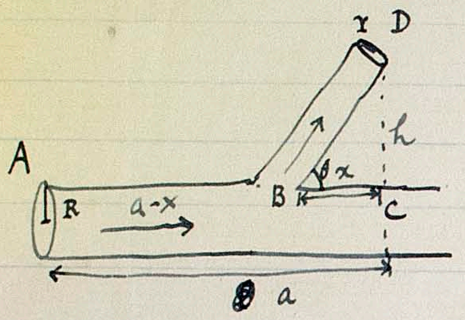
(Biometryヲ除キ、コレハ三年デ教ヘル).

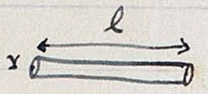
Morphology of blood vessels.

Burns' Biophysics

1. 動脈ノ枝ノナス角. (~~John Hunter~~).

AD間ノ血壓ノ差ヲ min. ナラシメル.





血壓ノ差 $= k . \frac{l}{r}$ (Hess's law)

AD間ノ血壓ノ差 $= k \left(\frac{AB}{R} + \frac{BD}{r} \right)$.

$$= k \left(\frac{a-x}{R} + \frac{\sqrt{h^2+x^2}}{r} \right)$$

$-\frac{1}{R} + \frac{2x}{2r\sqrt{x^2+h^2}} = 0$ $\qquad \therefore \frac{r}{R} = \frac{x}{\sqrt{x^2+h^2}} = \cos\theta$

故ニ

(1) Rガ一定ナルトキ, rガ等シケレバ θモ一定.

(2) $\frac{r}{R}$ガ甚ダ小ナレバ $\cos\theta$ガ小, 故ニ $\theta \to 90°$

(3) $\frac{r}{R} \to 1$ ナレバ $\cos\theta \to 1$ $\therefore \theta \to 0$

2. Dissociation of Oxyhaemoglobin.

Hb ノ saturation percentage $= y = 100 \frac{C_{(HbO_2)}}{C_{(Hb)} + C_{(HbO_2)}}$

Oxygen ノ concentration $= C_{(O_2)} = x$.

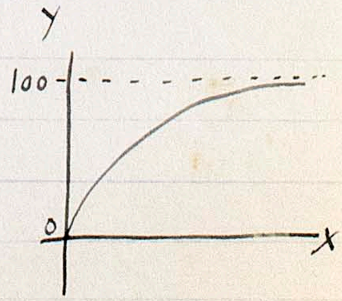
xトyトノ関係ヲ求ム.

$$Hb + O_2 \rightleftarrows HbO_2$$

平衡ノ場合ニハ $\frac{C_{(HbO_2)}}{C_{(Hb)} \cdot C_{(O_2)}} = K$.

$\therefore \frac{C_{(HbO_2)}}{C_{(Hb)}} = Kx$. $\qquad y = 100 . \frac{\frac{C_{(HbO_2)}}{C_{(Hb)}}}{1 + \frac{C_{(HbO_2)}}{C_{(Hb)}}}$

$\therefore y = 100 \frac{Kx}{1+Kx}$.



Hyperbola

3. Determination of the order of the chemical reaction.

分解スル場合

1. $H_2O_2 = H_2O + O$ ~~mono~~ unimolecular reaction

ナルカ $2H_2O_2 = 2H_2O + O_2$ bimolecular 〃

ナルカ.

此ノ場合ハ uni ナリ.

$$\frac{dx}{dt} = K(a-x)^n. \qquad n \text{ order of reaction.}$$

$a-x=C$, $\frac{dx}{dt} = -\frac{dC}{dt}$, $\frac{dC}{dt} = -KC^n$.

$$\int \frac{dC}{C^n} = -Kt + A.$$

$n=1$. $-\log C = \cancel{A-Kt}$ $Kt - A$

$n=2$ $+\frac{1}{C} = \cancel{A-Kt}$ $Kt - A$

$n=3$ $+\frac{1}{2C^2} = \cancel{A-Kt}$ $Kt - A$

n $\frac{1}{(n-1)C^{n-1}} = Kt - A.$

第一法.

$K = \frac{1}{t}\log\frac{a}{a-x}$

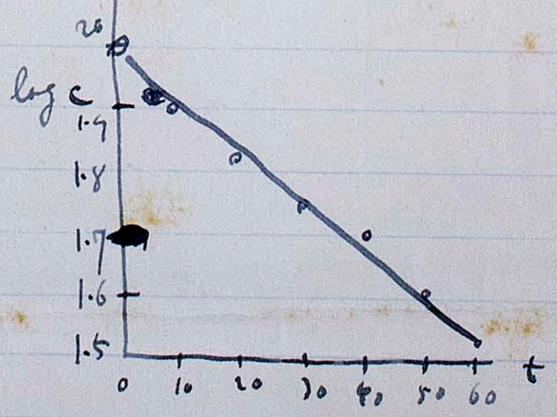
$K = \frac{1}{t}\left(\frac{1}{a-x} - \frac{1}{a}\right)$ コレガ const. トナラネバ

......

第二法. graphical.

Ex. 1. Madsen and Famulener. vibriolysin 加水分解

t (分)	C	log c
0	100	2
10	78.3	1.894
20	67.6	1.830
30	59.3	1.775
40	49.8	1.699
50	40.8	1.610
60	34.4	1.537



故ニ unimolecular.

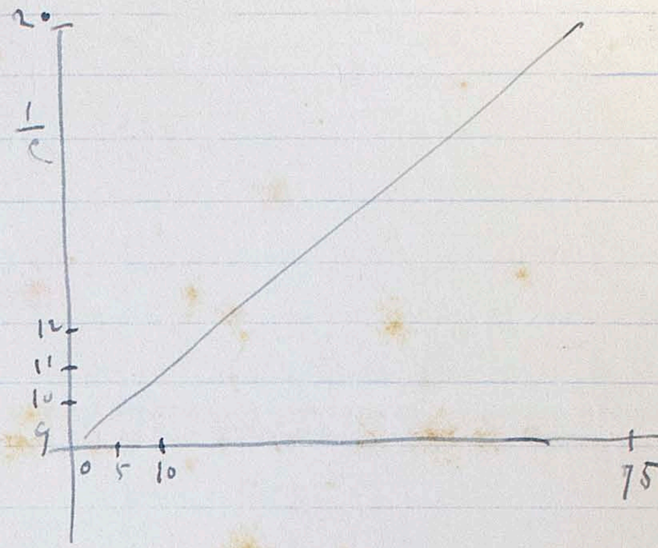
Ex. 2. Madsen and Walbum.

Casein ヲ trypsin デ分解サセ (catalysis).

casein ノ量ハ窒素デハカル.

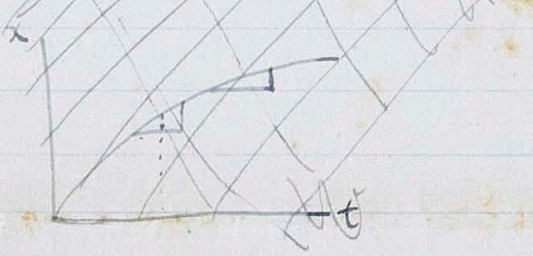
牛乳ノ量ヲ

t (hour)	C (N)	$\frac{1}{c}$
0	0.11	9
(0.25	0.108	9.2
2.5	0.102	9.8
6	0.1	10.0
11	0.096	10.4
24	0.076	13.0
33	0.07	14.3
48	0.06	16.6
(72	0.049	20.0



bi-molecular

第三法 slope method.



4. Number of micro-organisms.

食物ヲ養分ニヨツテニオク,

a = 食物ノ元ノ濃度.

$a-x$ = 時 t ニ於ケル食物ノ濃度

y_0 = 初メノ bacteria ノ数.

y = 時 t ニ於ケル bacteria ノ数.

(x ト y トハ正比例スルト assumeスレバ $y = cx$. ~~$y = x$~~ $x = y$. ~~$x = my$~~).

$$\frac{dy}{dt} = k\,\underset{\text{Number of bacteria}}{y}\,\underset{\text{Concentrat. of food}}{(a-x)}.$$

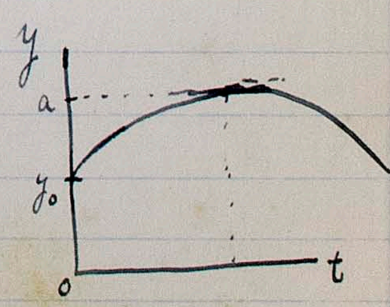
$$\frac{dy}{dt} = ky(a-y).$$

$$kt + C = \int \frac{dy}{y(a-y)} = \int \frac{1}{a}\left[\frac{1}{y} + \frac{1}{a-y}\right] dy = \frac{1}{a}\left[\log y - \log(a-y)\right].$$

$$\cancel{\log y} \quad \log \frac{y}{a-y} = akt + aC.$$

$t = 0$ ノトキ $\quad y = y_0, \qquad \log \frac{y_0}{a-y_0} = aC.$

$$\log \frac{y}{a-y} = akt + \log \frac{y_0}{a-y_0}.$$



$$\log \frac{(a-y_0)\,y}{y_0(a-y)} = akt$$

$$\frac{(a-y_0)\,y}{y_0(a-y)} = e^{akt}, \qquad \cancel{y = y_0 a e^{akt}} \qquad y = \frac{a}{1 + \frac{a-y_0}{y_0} e^{-akt}}.$$

M'Kendrick and Kesava Pai (1911)

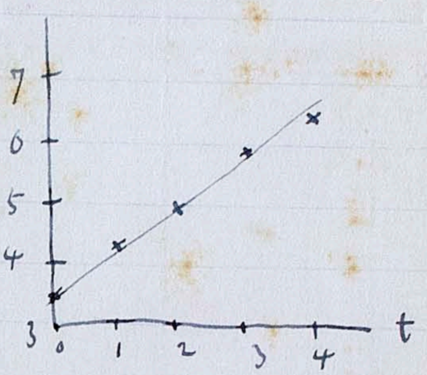
Bacillus coli 大腸菌.

t (hour)	0	1	2	3	4	
y	2 850	17 500	105 000	625 000	2250 000	100 000 000 ノト

発達止ムニ

$$\begin{cases} y_0 = 2850 \\ a = 100\ 000\ 000 \end{cases}$$

t	0	1	2	3	4
$\log_{10} \frac{y}{a-y} \doteqdot \log_{10} y$	3.46	4.24	5.02	5.90	6.35



$$\log_{10} \frac{y}{a-y} = ak \log_{10} e \cdot t + \log_{10} \frac{y_0}{a-y_0}.$$

$$ak \log_{10} e \doteqdot 0.8$$

bacteria ノ数ガ二倍ニナル時間.

$$\log_{10} \frac{2y}{a-2y} = ak \log_{10} e \cdot t' + \log_{10} \frac{y_0}{a-y_0}.$$

$$\log_{10} 2 + \log_{10} \frac{y}{a-2y} = 2k \log_{10} e\, t' + \cdots.$$

$$\therefore \quad \log_{10} 2 \doteqdot ak \log_{10} e \cdot (t'-t)$$

$$t'-t = \frac{\log_{10} 2}{ak \log_{10} e} = \frac{0.301}{0.8} \text{ hour}$$

$$= 22.5 \text{ min (分)}$$

5. Physiology of growth.

(Robertson, Child physiology.)

$$\log_{10} \frac{x}{341.5 - x} = K(t - 1.66)$$

t ハ 生レテカラノ月数. (−, +)

x ハ 体重 ~~oug~~ (オンス) ounce

9 個月マデニ適用.

t	x	K
−0.75	111	0.132
−0.42	117	0.136
−0.08	127	0.131
0	127	0.136
+0.25	137	0.123
+0.58	145	0.122
+0.92	146	0.171
	mean	0.136

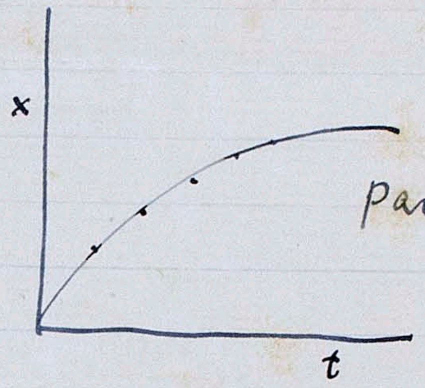
Lemon ノ成長

[illegible] K_1, K_2, K_3.

謬略.

1. Pepsin digestion (Sjöquist ノ研究).

t (hour)	x percent
0	0
2	10.5
4	16.41
6	19.93
8	22.68
9	24.00
12	27.04
16	30.36
20	33.68



parabola.

$$x = \frac{1}{K}\sqrt{t}.$$

$$(x = 8\sqrt{t}).$$

2. Decomposition of tetanolysin by means of peptone (Madsen and Walbum).

order of reaction = 2.

t (hour)	$C_{tetanolysin}$
0.5	47.7
1	39.7
2	30.3
4	22.3
6	18.1
8	17.0

6. Law of digestion.

$x =$ ~~quantity~~ ノ消化サレタ量

$x = c\sqrt{t}$.   Schütz-Borissoff law

$x^2 = c^2 t$.

$2x\frac{dx}{dt} = c^2$. $\frac{dx}{dt} = \frac{c^2}{2}\cdot\frac{1}{x}$ 故ニ t時ノ velocity ハ 消化サレタ量ニ 逆比例スル.

Theory of Arrhenius

$$\frac{dx}{dt} = K\cdot\frac{1}{x}(a-x)$$

$$\int\frac{x\,dx}{a-x} = kt + C. \qquad \frac{x}{a-x} = \frac{a}{a-x} - 1$$

$$-a\log(a-x) - x = Kt + C.$$

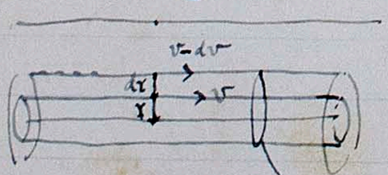
$t=0$ ノトキ $x=0$ $\qquad -a\log a = C.$

$$-a\log(a-x) - x = Kt - a\log a.$$

$$a\log\frac{a}{a-x} - x = Kt.$$

t (hour)	Protein digested by means of pepsin (percent)	$K = \frac{1}{t}\left[a\log\frac{a}{a-x} - x\right]$ $(a=1)$	$c = \frac{x}{\sqrt{t}}$
2	10.5	3.0	7.5
4	16.4	3.8	8.2
6	19.9	3.8	8.1
8	22.7	3.8	8.0
12	27.0	3.7	7.7
16	30.4	3.6	7.6
20	33.7	3.7	7.5
32	40.0	3.4	7.1
48	45.1	3.2	6.5
64	50.8	3.1	6.3
96	57.4	2.8	5.9

7. Flow of a viscous liquid through a tube.



粘性

管ノ方向ニ働ク力 (単位面積ニ就キテ) $\quad \eta\frac{-dv}{dr}$ $\qquad \eta$ = coefficient of viscosity.

$$-2\pi r l\cdot\eta\frac{dv}{dr} = \pi r^2 p \qquad p\text{ ,, pressure difference.}$$

$$-2l\eta\frac{dv}{dr} = rp.$$

$$-2l\eta v = p\frac{1}{2}r^2 + C. \qquad r=R \text{ ノトキ } v=0$$

$$\therefore\ 0 = \frac{p}{2}R^2 + C.$$

$$-2l\eta v = \frac{p}{2}(r^2 - R^2)$$

$$\therefore\ v = \frac{p(R^2-r^2)}{4l\eta}.$$

$2\pi r\,dr.$

単位時間ニ通ル体積 $\qquad dV = v\cdot 2\pi r\,dr$

$$V = \frac{p(R^2-r^2)\cancel{2}\pi r\,dr}{2\cancel{4}l\eta}$$

$$\nabla = \int_0^R \quad dr = \frac{p\pi}{2l\eta}\int_0^R r(R^2-r^2)\,dr$$

$$= \frac{p\pi}{2l\eta}\left(\frac{R^4}{2}-\frac{R^4}{4}\right) = \frac{p\pi R^4}{8l\eta}.$$

$\cancel{\nabla = \frac{\pi p R^4}{8\nabla l}}$ ~~Pu~~ Poisseuille

$$p = \frac{8\nabla l\eta}{\pi R^4} \quad \text{dyne}/\text{cm}^2$$

Brodie	尿管	$l = 1.2^{\text{cm}}$	$R = 6 \quad \frac{1}{1000}$ mm
		0.9	5
		0.9	4.5
		0.2	9
		2.2	8

$\eta = 719 \times 10^{-5}$ (35°ニテ)

$\nabla = 1^{\text{c.c}}$ per ~~min. (一分間)~~ Minute