微積分練習

1. $f(x)=x^{\frac{2}{3}}-x^{-\frac{2}{3}}$ ナルトキ $f(8)$ ヲ計算セヨ.

2. 次ノ函数ノ圖ヲ畫ケ (Graph)

I. $y=-x+2.$

II. $y=x^2$

3. 温度 t ニ於テ水 100 gram ノ中ニ溶解スル ~~Kalium Sulfat~~ Kaliumsulfat K_2SO_4 ノ量 (gr.) λ ハ

t	0°,05	4°,32	11°,41	18°,38
λ	7,36	8,16	9,49	10,81

λ ヲ t ノ函数トシテ圖示セヨ. (Graph)

4. ~~$x^3-xy+3=5y$~~ $f(x)=\log_{10}x$ ナルトキ次ノ式ヲ證明セヨ.
~~ナルトキ, y ヲ x ノ函数トシテ表ハセ.~~

$f(x)+f(y)=f(xy).$

$f(x^2)=2f(x).$

5. x ガ 0 ニ近ヅクトキ次ノ Funktion ハ Grenz ヲ有スルカ. 若シ Grenz ヲ有スルトキハ之ヲ求ム. ソノ Grenz ト $f(0)$ トヲ比較セヨ

I. $f(x)=x^m$ (m ハ正ノ整数 又ハ分数)

II. $f(x)=\dfrac{x^2}{x}$

III. $f(x)=\sin x.$

IV. $f(x)=$ ~~$\cos x$~~ $x\cos x.$

V. $f(x)=$ ~~$x\cos x$~~ ~~1000~~ $10^{10}x-5$

6. ~~I. $\lim_{x=3}\dfrac{x^2-9}{x-3}$~~, II. $\lim_{x=3}\dfrac{x^2-9}{(x-3)^2}$

7. $$\lim_{x=\infty}\frac{a_0x^n+a_1x^{n-1}+\dots+a_{n-1}x+a_n}{b_0x^m+b_1x^{m-1}+\dots+b_{m-1}x+b_m}\quad\begin{pmatrix}a_0\neq 0\\ b_0\neq 0\end{pmatrix}$$

8. $f(x) = \cot x$ ハ如何ナル点ニ於テ unstetig ナルカ.
図ヲ画ケ.

9. ~~[struck]~~ ヨリ次ノ Differentialquotient ヲ求ム

$$y = -3x^2 + 2x + 5.$$

ソノ ~~曲線~~ (Kurve) ~~[struck]~~ ノ ~~[struck]~~ Tangente ノ中ニテ x-Achse
ニ parallel ナルヲ求ム.

10. 化学反応ノ速サヲ数学的ニ表ハセ.

試験

11. $$\lim_{x=\infty} \frac{10000x^2 + 1}{0{,}00001\,x^4 + 0{,}0001\,x^3 - 100000\,x + 2}$$

12. Kurve $4y = x^4 - 4x - 1$
ノ Punkt $(1, -1)$ ニ於ケル ~~[struck]~~ Steigung ヲ求ム.

$4\frac{dy}{dx} = 4x^3 - 4$
$\frac{dy}{dx} = x^3 - 1$
$4y = x^4 - 4x - 1$
$4y = 1 - 4 - 1 = -4$

13. $y = (x+1)^2(2x-3)$ $\qquad \frac{dy}{dx} = 6x^2 + 2x - 4$

~~14. $y = \frac{5+x}{5-x}$ $\frac{a^2 - x^2}{a^2 + x^2}$ $\qquad \frac{dy}{dx} = -\frac{4a^2x}{(a^2+x^2)^2}$~~

~~15.~~ 14. $y = \frac{5 + 3x + x^2}{5 - 3x + x^2}$ $\qquad \frac{dy}{dx} = \frac{6(5 - x^2)}{(5 - 3x + x^2)^2}$.

15. $y = \frac{x}{x + \sqrt{1 + x^2}}$ $\qquad \frac{dy}{dx} = \frac{(\sqrt{1+x^2} - x)^2}{\sqrt{1+x^2}}$

16. $(x^2 + y^2)^2 - 2c^2(x^2 - y^2) = 0$ $\qquad \frac{dy}{dx} = -\frac{x(x^2 + y^2 - c^2)}{y(x^2 + y^2 + c^2)}$.

17. $y = \left(\frac{1}{\sqrt{x}} + 2\right)(x - \sqrt{x})$ $\qquad \frac{dy}{dx} = 2 - \frac{1}{2\sqrt{x}}$.

1. 次ノ曲線ノ図ヲ画ケ

$$y=\frac{1}{1-x^2}$$

2. 極大極小

~~(I) $y=\sin 2x-x$,~~

(II) $y=\sin x+\cos x$

3. 微分セヨ

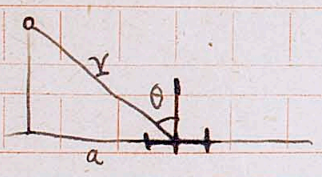
I. $y=\sqrt{\dfrac{1+\cos x}{1-\cos x}}$, II. $y=tg\left(\dfrac{\pi}{4}-\dfrac{x}{2}\right)$

III $\sin 2x=2\sin x\cos x$,

IV. $y=\mathrm{arc}\,tg\,\dfrac{\sqrt{a^2-b^2}\,\sin x}{b+a\cos x}$. $\left(\dfrac{\sqrt{a^2-b^2}}{a+b\cos x}\right)$

4.

Licht 強サ ハ $\dfrac{\cos\theta}{r^2}$ = 比例ス.



a ヲ与ヘテ 最モ明ルクスベキ 光ノ高サヲ 求ム.

5. $\dfrac{dy}{dx}$ ヲ求ム. I. $y=\log\sin x$, II $y=x^x$

$\left[\ \log y=x\log x,\quad \dfrac{dy}{y\,dx}=\log x+1,\quad \therefore \dfrac{dy}{dx}=x^x(1+\log x)\right.$

III. $y=x^{\sin x}$, IV. $y=\log tg\left(\dfrac{x}{2}+\dfrac{\pi}{2}\right)$.

~~I. $y^3=a^2(x-y)$, II. $2x^2-3xy-y^2=1$,~~

~~III. $x^2y-y^2x+ax^2-aby=0$.~~

6. $1+x+x^2+\cdots+x^n=\dfrac{1-x^{n+1}}{1-x}$

ト[illegible]コトヨリ $1+2x+3x^2+\cdots+nx^{n-1}=\dfrac{1-(n+1)x^n+nx^{n+1}}{(1-x)^2}$

ヲ証明セヨ. コレヨリ $1+2^2x+3^2x^2+\cdots+n^2x^{n-1}$ ノ和ヲ求ム.

$x+2x^2+3x^3+\cdots+nx^n$

1. Differenziere folgende Funktionen:

1. $y = a^2 \arcsin \frac{x}{a} + x\sqrt{a^2 - x^2}$

2. $y = \log\left(x + \sqrt{x^2+a^2} - x\sqrt{x^2+a^2}\right)$

3. $y = 10^{x^2}$

4. $y = (\sin x)^{\sin x}$

5. $y = \log(\log x)$

6. $y = \log \frac{\sqrt{a+bx} - \sqrt{a}}{\sqrt{a+bx} + \sqrt{a}}$

7. $x = \frac{c}{2}\left(e^{\frac{y}{c}} + e^{-\frac{y}{c}}\right)$.

8. $y = x^{\log x}$

9. $y = \operatorname{arc\,cot}(n \sin x)$

10. $y^3 = a^2(x - y)$

$y = \operatorname{arc\,tg}\sqrt{\frac{1-\cos x}{1+\cos x}}$

11. $2x^2 - 3xy - y^2 = 1$

12. $x^3 - xy + y^4 = 0$

注意

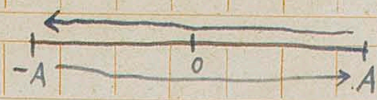
1. $\frac{d \log x}{dx} = \frac{1}{x}$.

$\frac{d \log 10}{dx} = 0$.

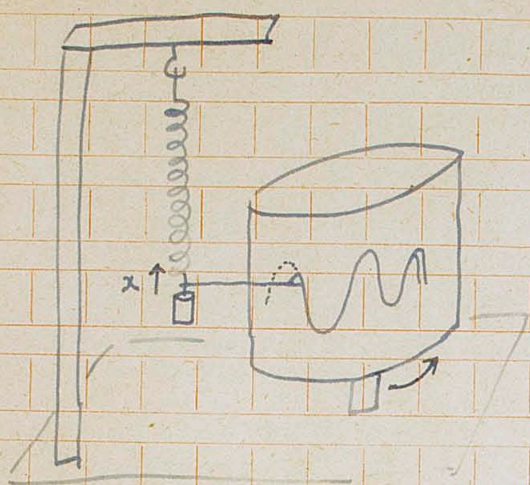
2. $\frac{df(ax)}{dx} = a \cdot \left(\frac{df(x)}{dx}\right)$

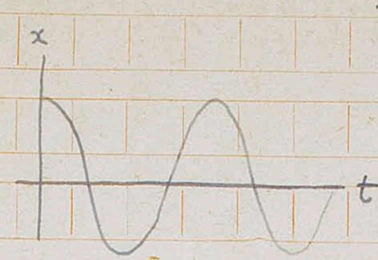
1. $x = A\cos kt$ ungedämpte Schwingung
simple harmonic motion 單弦運動

t	0	$\frac{\pi}{2k}$	$\frac{\pi}{k}$	$\frac{3\pi}{2k}$	$\frac{2\pi}{k}$	…
x	A	0	$-A$	0	A	…



$T = \frac{2\pi}{k}$ Period. A Amplitude 振幅





Geschwindigkeit v u. Beschleunigung α ヲ求ム.

$$\alpha = -k^2 x.$$

[角ノ小ナル振子ノ運動等]

2. ~~$\frac{d^2x}{dx^2}$ ヲ求ム~~

$y = a\cos(\log x) + b\sin(\log x)$ ナルトキハ $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$ Differentialgleichung.

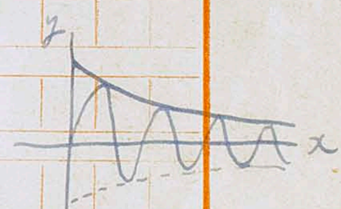
~~I. $y = \sin ax$, II. $y = e^{ax}$, III. $y = \log x$.~~ ナルカヲ[illegible]

3. 曲線 ~~$y = 1 + \frac{1}{x}$~~

$$y = e^{-t}\sin t$$

[旋体体ノ回振動 / 抵抗ヲ加ヘル マサツ等]

ヲ画ケ. (Max. Min ヲ求ム)



4.

1. $\int \frac{x\,dx}{x^2-a^2}$ $\qquad$ $\frac{1}{2}\log(x^2-a^2)+C.$

2. $\int (2x+1)^3\,dx$ $\qquad$ $\frac{1}{8}(2x+1)^4$

3. $\int \frac{3x+2}{x^2-x-2}\,dx$ $\qquad$ $\frac{8}{3}\log(x-2)+\frac{1}{3}\log(x+1)$

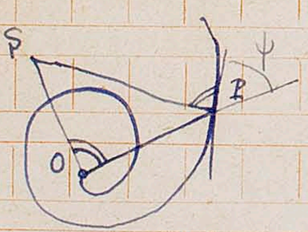
4. $\int \frac{x^2+1}{x^3-x}\,dx$ $\qquad$ $\log\frac{x^2-1}{x}$

5. $\int \sin^2 x\,dx$ $\qquad$ $-\frac{1}{4}\sin 2x+\frac{x}{2}$

1. Kardioide $r=a(1-\cos\theta)$ ノ Gleichg ヲ rechtwinklige Koordinaten ニテ表ハセ.

2. $r=\sin 5\theta$ ヲ画キ, ソノ面積ヲ求ム

3. Archimedische Spirale $r=a\theta$

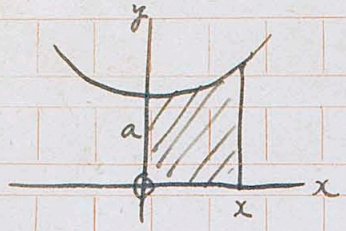


ニ於テ $OS=$ Konst. ナルコトヲ証明セヨ

4 β. $\operatorname{tg}\psi=$ Konst. ナル総テノ曲線ヲ求ム.

$$\left[\operatorname{tg}\psi=\frac{r\,d\theta}{dr}\right]$$

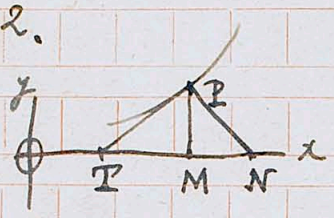
1. $y = \frac{a}{2}\left(e^{\frac{x}{a}} + e^{-\frac{x}{a}}\right)$



$MN = k$. 一曲線をおく,

$Y - y = \frac{dx}{dy}(X - x)$. $\left(-\frac{dx}{dy}\right)$.

2.



$-y\frac{dy}{dx} + x = X$.

$\left[\left(-\frac{y}{\frac{dy}{dx}} + x\right) - x\right]^2 + (y-0)^2 = \frac{y^2}{\left(\frac{dy}{dx}\right)^2} + y^2 = k^2$.

$y^2 = (k^2 - y^2)\left(\frac{dy}{dx}\right)^2$

$-y\frac{dy}{dx} + x = X$.

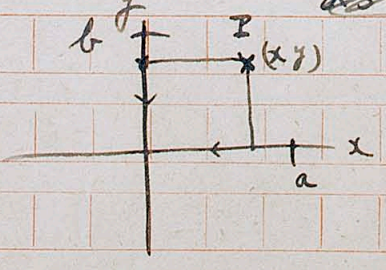
$\frac{dy}{dx} = \pm \frac{y}{\sqrt{k^2 - y^2}}$

$y\frac{dy}{dx} = k$.

$C + kx = y^2$.

3.

$ds^2 = dx^2 + dy^2$



$\begin{cases} \frac{d^2x}{dt^2} = -k^2 x \\ \frac{d^2y}{dt^2} = -k^2 y \end{cases}$

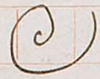
$t=0,\ x=a,\ y=b$

$\frac{dx}{dt}=0,\ \frac{dy}{dt}=0$.

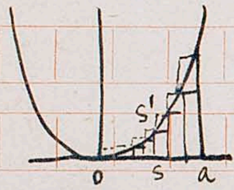
$\begin{cases} x = a\cos kt \\ y = b\cos kt. \end{cases}$

$t=0,\ x=0,$

$\frac{dx}{dt} = ka$.

1. 質点ガ力ノ中心ヲ極トセル Logarithmische spirale  ヲ画クトキ, ソノ力ノ法則如何.

2. $y=f(x)=x^2$.



$$s=\frac{a^3}{n^3}\left[0^2+1^2+2^2+\cdots+(n-1)^2\right]$$

$$s'=\frac{a^3}{n^3}\left[1^2+2^2+\qquad+n^2\right]$$

次ニ $1^2+2^2+\cdots+n^2=\frac{1}{3}(n+1)\left(n+\frac{1}{2}\right)n$. ヲ証明セヨ.

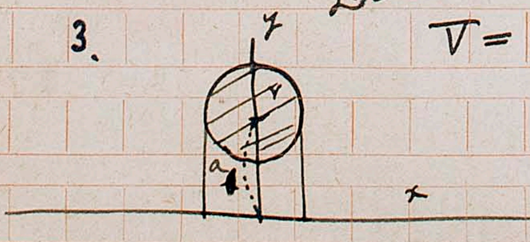
最後ニ $A=\int_0^a x^2\,dx=\frac{1}{3}a^3$ ナルコトヲ証明セヨ.

1. $\frac{d}{dx}\int_a^x f(x)\,dx=\ ?$

2. $\int_{-\infty}^{x}\frac{dx}{x^2}$

Das Volumen des Kreisringes ヲ求ム

3. $V=4\pi a\int_{-r}^{r}\sqrt{r^2-x^2}\,dx=2\pi^2 r^2 a$.



$x^2+(y-a)^2=r^2$ $\qquad y=a\pm\sqrt{r^2-x^2}$

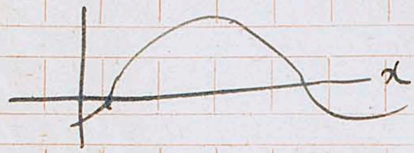
$$V=V_1-V_2=\pi\int_{-r}^{r}\left[a+\sqrt{r^2-x^2}\right]^2dx$$

$$-\pi\int_{-r}^{r}\left[a-\sqrt{r^2-x^2}\right]^2dx$$

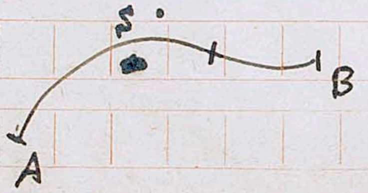
$$=4a\pi\int_{-r}^{r}\sqrt{r^2-x^2}\,dx.$$

1. $\int e^x \cos(e^x)\,dx$

2. $y = x - x^4$ ガ x 軸ノ周リニ回転スルトキ, x 軸ノ上方ニアル部分ニヨリテ生成セラルル体積ヲ求ム.



3. 長サ a ナル針金アリ, ソノ一点ノ密度ハ, 一端 A ヨリソノ点ニ到ル長サ s ニ比例ス. コノ針金ノ質量ヲ求ム.



1. 次ノ計算ハ誤リナリヤ

$1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}+\cdots = 1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\cdots$ ココニ誤アリ

$-2\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{6}+\cdots\right)$

$= 1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\cdots\cdots-\left(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\cdots\right)$ ココハ誤モ

$=0.$

$\left(=\frac{1}{2}\right)$

2. $\lim_{x=0} \frac{1-\cos x}{\sin^2 x}$

3. $\lim_{x=\infty} \frac{x^2-x}{1-x+\log x}$.

4. $y^2 = x(x-1)(x-2)$ ヲ x 軸ノ周リニ回転スルトキ, ソノ輪ニヨリテ生ズル体積ヲ求ム.

5. $f(x) > \phi(x)$ $\quad a < x < b$

ナラバ

$\int_a^b f(x)\,dx > \int_a^b \phi(x)\,dx$

(1) $xe^{-x^2} + (2xe^{-2x^2} - xe^{-x^2}) + (3xe^{-3x^2} - 2xe^{-2x^2}) + \cdots$

ハ収斂ナルコトヲ証明シ、ソノ和ヲ求ム.

[illegible]

$$\int_0^1 xe^{-x^2}dx + \int_0^1 (\quad)dx + \int_0^1 (\quad)dx + \cdots$$

ヲ求ム.

(2) $1+\sqrt{1}+3+\sqrt{3}+5+\sqrt{5}+\cdots$

ニ d'Alembert ヲ適用シ得ルカ.

(3) $1-\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}-\frac{1}{\sqrt{4}}+\cdots$

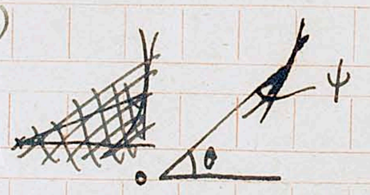
(1) 太陽ノ冷却

$$\frac{dT}{dt}=aT^3-bT \qquad T \text{ 絶対温度}$$

T ヲ t ノ Funktion トシテ求ムレバ

$$z=\frac{1}{T^2}=\frac{a}{b}+Ce^{2bt}.$$

(2)



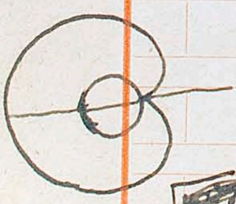
$\psi=\frac{\theta}{2}$ ナル如キ曲線ヲ求ム

$$tg\,\psi=\frac{r}{\frac{dr}{d\theta}}=tg\,\frac{\theta}{2}.$$

$$\frac{d\theta}{tg\frac{\theta}{2}}=\frac{dr}{r}. \qquad r = tg\frac{\theta}{2}\,\frac{dr}{d\theta}$$

$$\log\left(\sin\frac{\theta}{2}\right)^2=\log\frac{r}{c}.$$

$r=c(1-\cos\theta)$ Kardioide.



$\frac{dy}{dx}=\frac{-(2xy-k)\pm\sqrt{\cdots-4kxy}}{2x^2}$ (3)

$\eta-y=\frac{dy}{dx}(\xi-x)$ $\eta=0.$ $\xi=0$

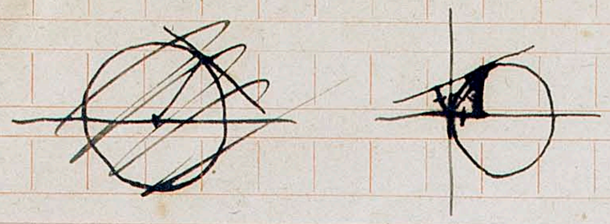
$-y\frac{dx}{dy}+x=\xi$

$+y-x\frac{dy}{dx}=\eta$

$+y^2-(2xy-k)\frac{dy}{dx}+x^2\left(\frac{dy}{dx}\right)^2=0.$

$-y^2\frac{dx}{dy}+2xy-x^2\frac{dy}{dx}=k$

(3)



$(\eta - y) - \frac{dy}{dx}(\xi - x) = 0.$

$$\frac{y - x\frac{dy}{dx}}{\sqrt{1+\left(\frac{dy}{dx}\right)^2}} = x.$$

$$y - x\frac{dy}{dx} = x\sqrt{\ }.$$

$$y^2 - 2xy\frac{dy}{dx} + x^2\left(\frac{dy}{dx}\right)^2 = x^2 + x^2\left(\frac{dy}{dx}\right)^2$$

$$y^2 - x^2 - 2xy\frac{dy}{dx} = 0.$$

$y = zx.$

$$\frac{dy}{dx} = x\frac{dz}{dx} + z.$$

$$z^2x^2 - x^2 - 2x^2z\left(z + x\frac{dz}{dx}\right) = 0.$$

$$+z^2x^2 + x^2 + ~~2x^2z^2~~ + 2x^3z\frac{dz}{dx} = 0.$$

$$z^2 + 1 + 2xz\frac{dz}{dx} = 0.$$

$$2xz\frac{dz}{dx} = 1 + z^2 \qquad \frac{2z\,dz}{1+z^2} = -\frac{dx}{x}$$

~~$-\frac{1}{3}\log(1-3z^2) = \log x + \log c$~~

~~$(1-3z^2)^{\frac{1}{3}} = cx.$~~

~~$c^3x^3 - 3c^3xy^2 = 1.$~~

$$\log(1+z^2) = -\log x + \log c.$$

$$1 + \frac{y^2}{x^2} = \frac{c}{x}.$$

$$x^2 + y^2 = cx.$$

(4)

$$\sin^2 x + \sin^2 x\cos^2 x + \sin^2 x\cos^4 x + \dots + \sin^2 x\cos^{2n} x + \dots$$

ハ收斂スルヤ否ヤ。若シ收斂スレバ、ソノ和ヲ求ム、

$v = 0,\quad x = 0, \pm\pi, \pm 2\pi, \dots$

$= 1\quad$ 他ノ値ニツキテハ

(5) $x^2 + \frac{x^2}{1+x^2} + \frac{x^2}{(1+x^2)^2} + \dots$

$x = 1 \quad 0$

$x \neq$

(6)

~~$1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$~~

$$1 + x + x^2 + x^3 + \dots = \frac{1}{1-x} = y$$